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Tau Zero: In the cockpit of a Bussard ramjet

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A Bussard ramjet is a relativistic spacecraft, fueled by fusion energy of cosmic matter that is collected during the flight. We derive the equation of motion of such a spaceship for a given mass density in space and the fusion mass defect. Two ramjet engine scenarios, where the thrust for propulsion is generated by emission of photons or acceleration of matter, are outlined. As long as not all collected matter is transformed into fusion energy, mass engines are superior to photon engines. If the collected matter is stopped by the spacecraft before fusion it may not reach relativistic terminal velocities. For an ideal ramjet, where no matter is stopped for the generation of energy for propulsion, endless acceleration and relativistic velocities may be obtained such that crossing the universe in a human lifespan would be possible. A journey along one space coordinate and the smallest possible radii of curves were evaluated. The results are compared to the plots in the novel "Tau Zero" by Poul Anderson.
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I. INTRODUCTION

Tau, the lead word in the novel "Tau Zero" by Poul Anderson,[1] refers to the reciprocal value of the Lorentz factor γ, that relates time intervals in two rest frames moving relative to each other. For an astronaut in a spaceship his clock is a factor of Tau slower than that in the frame of the universe in which he is moving. This means that the aging of the astronaut is slower than that of the ground control. In his novel, Anderson[1] adopted a ramjet as described by Bussard[2] for an interstellar trip, where the ramjet and its crew are doomed to almost endlessly accelerate. He describes a relativistic scenario where Tau approaches zero (or γ becomes very large). In the present paper, the Greek letter τ is the time coordinate in the spaceship that is related via Tau (i.e., $1/\gamma$) to the time coordinate of the cosmos.

In order to obtain high relativistic velocities Anderson chose the concept of a Bussard ramjet instead of a rocket.[3,4] While a rocket is initially loaded with all fuel for a trip, a ramjet collects interstellar matter (such as protons) through a magnetic intake funnel and uses them for fusion and propulsion. Serious limitations or the infeasibility of such a device were discussed, including energy loss by radiation and mass loss[2,5–7] or the structural strength of the construction of such a device.[8,9] These details, different fusion reactions, and how the released momentum would be redirected are technical problems[10–13] and are not treated here. In the following, we explore the physical limits and highlight the topic for pedagogical purposes.

Of course, a Bussard ramjet must also obey energy and momentum conservation of the spacecraft and the involved interstellar matter, where the mass defect from the fusion reaction is considered.[2,14] We will treat two scenarios for the engine producing the thrust for propulsion: the emission of photons or the acceleration of matter, which yield different accelerations in otherwise identical situations. Furthermore, if the matter that is used for propulsion has to be stopped on the ramjet prior to conversion into energy for propulsion, then no velocities above one light year per year in the spaceship may be obtained. If this constraint of friction is released in an ideal ramjet, a non-zero terminal acceleration is predicted.

For the description of the motion, we use proper quantities, as they are experienced by the astronauts in the cockpit of the ramjet: the proper time τ is the time measured by clocks in the spacecraft, the proper velocity w gives the distance $\mathrm{d}x$ traveled in the reference frame of the cosmos in a given interval of proper time $\mathrm{d}\tau$, and the proper acceleration a that affects the weight felt by the passengers of the spaceship.

A short overview of the formulation of kinematics and dynamics in proper quantities is given in Sec. II. The energy available for propulsion is discussed in Sec. III. Ramjets stopping the incoming matter are discussed in Sec. IV. The kinematics of ideal ramjets not stopping the incoming matter are treated in Sec. V. Section VI explores consequences with respect to travel times and maneuverability of an ideal ramjet. Finally, Sec. VII concludes the topic.

II. RELATIVITY WITH PROPER QUANTITIES

Velocity is generally described with the differential limit

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t}, \tag{1}$$

where $\mathrm{d}x$ is a change in position within the time step $\mathrm{d}t$. In Galilean spacetime, $\mathrm{d}t$ is the same in all frames of reference. In relativistic spacetime, this is not the case and a careful distinction of different operational definitions of velocity must be made.[15]

If both distance and time are measured in the same system of reference, we get the coordinate velocity $v = \beta c$, where β is the ratio between v and the speed of light c. If the time is measured in the rest system of the moving body we get the proper velocity

$$w = \frac{\mathrm{d}x}{\mathrm{d}\tau} = \omega c, \tag{2}$$

where ω is the ratio between w and the speed of light c. The proper velocity w is related to the coordinate velocity v by

$$w = \gamma v, \tag{3}$$

or with the dimensionless velocity ratios ω and β by

$$\omega = \gamma\beta, \tag{4}$$

where the dimensionless Lorentz factor γ is

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \sqrt{1+\omega^2}. \tag{5}$$

Without gravitation, the weight of a passenger with mass m is ma, where a is the proper acceleration[15–17]

$$a = \frac{1}{\gamma}\frac{\mathrm{d}w}{\mathrm{d}\tau}. \tag{6}$$

Using the proper quantities, the notation of classical dynamics can be recovered for relativistic dynamics, where

$$F = \frac{m}{\gamma}\frac{\mathrm{d}w}{\mathrm{d}\tau} \tag{7}$$

is the relativistic Second Law for the force F acting on the mass m, written in terms of proper acceleration a, or in terms of proper velocity and proper time. Correspondingly, the relativistic radial acceleration a_r in uniform circular motion is

$$a_\mathrm{r} = \frac{w^2}{r} = \frac{(\omega c)^2}{r}, \tag{8}$$

where $w = \omega c$ is the proper velocity on the circular trajectory of radius r.

From Eq. (7), we obtain for free particles the relativistic linear momentum, which reads familiar when written with the proper velocity,[16]

$$p = mw = m\omega c, \tag{9}$$

and finally, the total energy

$$E = \gamma mc^2, \tag{10}$$

where $E_0 = mc^2$ is the rest energy of the mass m.

In the following, we will omit the specifying term *proper* for the quantities of proper velocity $w = \omega c$, proper acceleration a, and proper time τ and we use the dimensionless proper velocity ratio ω.

III. ENERGY AND MOMENTUM FOR PROPULSION

The energy for propulsion of a Bussard ramjet stems from fusion, i.e., the mass defect of the collected matter. This energy is then used to change the momentum and accelerate the ramjet by exhaust of radiation or particles opposite to the direction of acceleration, such that the additional momenta in the exhaust and the ramjet cancel. In the following, we will determine the cosmic mass that may be collected for propulsion and describe energy and momentum in the rest frame of the universe (cosmic frame).

A. Collected mass

A Bussard ramjet collects cosmic matter with a density ρ over a receiver area A. Moving with a velocity ωc, it covers a given distance $\mathrm{d}x = \omega c\,\mathrm{d}\tau$ measured in the cosmic frame in a time interval $\mathrm{d}\tau$. Thus, the mass $\mathrm{d}m$ of cosmic matter collected in the time interval $\mathrm{d}\tau$ is

$$\mathrm{d}m = \rho A\,\mathrm{d}x = \rho A\omega c\,\mathrm{d}\tau, \tag{11}$$

where the mass defect $\epsilon\,\mathrm{d}m$ is transformed into the energy for propulsion $\epsilon\,\mathrm{d}mc^2$. At this point, we recall that for any known processes not involving antimatter, the mass defect coefficient ϵ is small. For example, the nuclear fusion reaction that transforms 4 protons into ^{4}He has a mass defect coefficient $\epsilon = 0.007$. In particular, if fusion of baryonic matter is used for propulsion with photons only, then $\epsilon = 1$ is impossible because this would violate conservation of the baryon number.

The thrust may be realized by exhausting the remainder of the received mass after fusion or by emitting electromagnetic radiation. These two engine concepts impose different performances in view of acceleration and terminal velocities for a given ϵ, and for $0 < \epsilon < 1$ a mass engine is always superior to a photon engine. This can be seen in an energy momentum diagram that describes the change of state of motion of the ramjet.

B. Energy-momentum-diagram

The total energy of a mass m depends on its rest energy mc^2 and its momentum p. Using Eqs. (5), (9), and (10) we obtain

$$E = c\sqrt{(mc)^2 + p^2}, \tag{12}$$

while the corresponding energy momentum relation for massless ($m = 0$) particles such as photons reduces to

$$E = pc. \tag{13}$$

Comparison of Eqs. (12) and (13) indicates that for a given energy E the momentum is largest for massless particles. Still, as we will see in the following, a matter engine in a ramjet performs better than a photon engine for all $0 < \epsilon < 1$. This fact also applies for a rocket, where not all fuel is accelerated to the exhaust velocity.[4]

In Fig. 1, energy and momentum of the ramjet are shown in units of the ramjet with mass M (i.e., Mc^2 and Mc, respectively). The ramjet mass does not change if the infinitesimal mass of cosmic matter $\mathrm{d}m$ collected, partly accelerated, and ejected in the time interval $\mathrm{d}\tau$ is neglected. An acceleration process changes the momentum of the ramjet by $\mathrm{d}p$, its energy by $\mathrm{d}E$, and all states of motion of the ramjet obey Eq. (12), i.e., they lie on the solid line labeled M.

Conservation of momentum requires a compensation of $\mathrm{d}p$ by $-\mathrm{d}p$ in the engine exhaust. This may be realized by emission of massive particles or photons, which have to obey their corresponding energy-momentum relations as well. A ramjet harvests the propulsion energy $\epsilon\,\mathrm{d}mc^2$ in the time interval $\mathrm{d}\tau$. Therefore, we mark the addition of the propulsion energy as a transition to a virtual state 2. If there is friction, the momentum of the virtual state gets reduced with respect to the momentum of state 1. In Fig. 1, this is shown in connecting point 1, the initial state of motion, with point 2. Obviously, this state 2 does not lie on the solid line, which

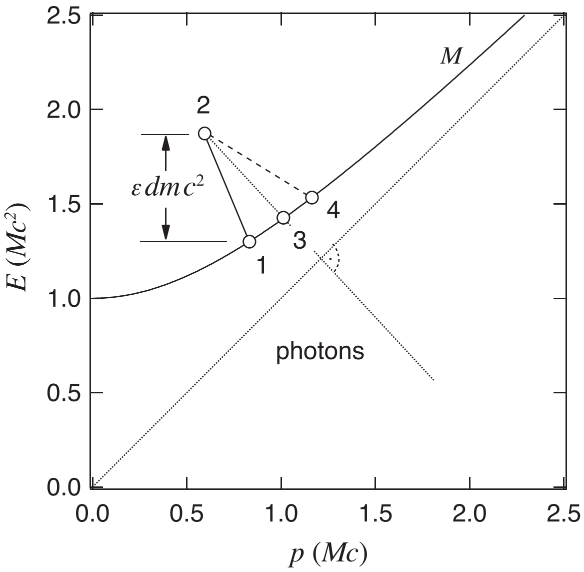

Fig. 1. Energy momentum diagram. Solid line: massive particle (ramjet) with mass M; dotted lines: photons. The picture refers to the rest frame in which E and p are measured. Propulsion promotes the ramjet within the time interval $\mathrm{d}\tau$ from state 1 into state 3 (photons) or state 4 (matter) on the solid line M. The propulsion energy $\epsilon\ \mathrm{d}mc^2$ adds to state 1 into a virtual state 2. If there is friction, the momentum of the virtual state gets reduced with respect to the momentum of state 1. The momentum and energy of the exhaust have to be subtracted from state 2 such that the result lies on the solid line. Since $|\mathrm{d}E/\mathrm{d}p| < c$ for massive particles, a matter engine may produce for a given $\epsilon\ \mathrm{d}m$ more propulsion momentum than a photon engine.

means that another process, such as the exhaust, must provide momentum and consume energy such that the sum of energy and momentum of state 2 and those of the exhaust lie on the solid line M again. We find $\mathrm{d}p$ in subtracting the exhaust energy and momentum from state 2, such that we hit the solid line. For a photon engine, this is particularly simple: $\mathrm{d}E_\mathrm{p}/\mathrm{d}p_\mathrm{p} = -c$ and we get the intersection of a line with slope $-c$ across state 2 with the solid line at state 3. For a matter engine $|\mathrm{d}E_\mathrm{m}/\mathrm{d}p_\mathrm{m}|$ is always smaller than c, and correspondingly $\mathrm{d}p$ is, for a given propulsion energy, always larger for a matter engine as compared to a photon engine. This statement is correct for $0 < \epsilon < 1$, while for $\epsilon = 0$ no thrust may be produced, and for the most efficient case $\epsilon = 1$ all incoming matter would be transformed into photons.

IV. RAMJETS STOPPING THE INCOMING MATTER

We consider a ramjet that moves through the cosmic matter. Following the scenario in Fig. 1 the jet accelerates in a time interval $\mathrm{d}\tau$ from a momentum $p_1 = M\ \omega_1 c$ to $p_3 = M\ \omega_3 c$ for a photon engine or to $p_4 = M\ \omega_4 c$ for a matter engine. For this, the collected mass is stopped in the ramjet and then the fraction ϵ of the collected cosmic matter is transformed via fusion into energy that is used to accelerate the ramjet. This inelastic collection process may be considered as friction. Furthermore, as we see from Fig. 1, only part of the propulsion energy $\epsilon\ \mathrm{d}mc^2$ increases the kinetic energy of the ramjet; the rest is found in the exhaust.

A. Photon engine

From Fig. 1, we find the energy balances

$$E_3 = E_2 - E_\mathrm{p} = E_1 + \epsilon\, \mathrm{d}m\, c^2 - E_\mathrm{p}, \tag{14}$$

where E_p is the energy of the exhausted photons. Meanwhile, the momentum balance is

$$p_3 = p_1 + p_\mathrm{p} - \mathrm{d}m\, \omega_1 c \tag{15}$$

where p_p is the momentum of the photons transferred to the ramjet and $-\mathrm{d}m\omega_1 c$ the momentum loss due to the stopping of $\mathrm{d}m$. Putting $p_\mathrm{p} = E_\mathrm{p}/c$ into Eq. (15) we can solve for E_p and get from Eq. (14)

$$(\gamma_3 - \gamma_1)M - (\epsilon - \omega_1)\, \mathrm{d}m + M(\omega_3 - \omega_1) = 0. \tag{16}$$

For a ramjet with a photon engine stopping the collected matter, where $\gamma_3 - \gamma_1 = 0$ and $\omega_3 - \omega_1 = 0$, the expression for the terminal velocity ratio $\omega_{\mathrm{p},\infty}$ of a photon engine follows from the second term in Eq. (16)

$$\omega_{\mathrm{p},\infty} = \epsilon. \tag{17}$$

The dashed line in Fig. 2 shows the terminal velocity ratio $\omega_{\mathrm{p},\infty}$ as a function of ϵ, of a ramjet with a photon engine stopping the incoming matter.

B. Mass engine

Again, from Fig. 1 we find the energy balances

$$E_4 = E_2 - E_\mathrm{m} = E_1 + \epsilon\, \mathrm{d}m\, c^2 - E_\mathrm{m}, \tag{18}$$

where E_m is the kinetic energy of the exhausted matter. With $E_\mathrm{m} = (\gamma_\mathrm{m} - 1)\,(1-\epsilon)\, \mathrm{d}m\, c^2$, Eq. (18) can be rewritten as

$$(\gamma_4 - \gamma_1)\, M - [1 - \gamma_\mathrm{m}(1-\epsilon)]\, \mathrm{d}m = 0, \tag{19}$$

where $\gamma_\mathrm{m} = \sqrt{1 + \omega_\mathrm{m}^2}$ is the Lorentz factor for the exhausted matter. With $\gamma_4 - \gamma_1 = 0$, it follows that the second term in Eq. (19) must be zero at the terminal velocity of a ramjet with a matter engine that stops the collected matter. The Lorentz factor γ_m follows from p_m in the momentum balance

$$p_4 = p_1 + p_\mathrm{m} - \mathrm{d}m\, \omega_1 c, \tag{20}$$

where $p_\mathrm{m} = (1-\epsilon)\ \mathrm{d}m\omega_\mathrm{m} c$ is the exhaust momentum transferred to the ramjet. The corresponding expression for the terminal velocity ratio $\omega_{\mathrm{m},\infty}$ of a ramjet with a matter engine stopping the incoming matter is then

$$\omega_{\mathrm{m},\infty} = \sqrt{\epsilon(2-\epsilon)}. \tag{21}$$

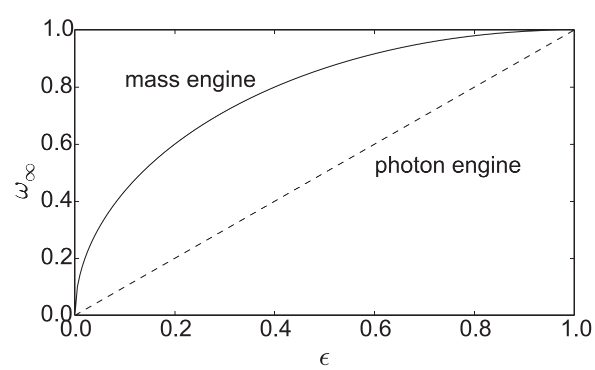

Fig. 2. Terminal velocity ratios ω_∞ of a Bussard ramjet stopping all incoming matter with a photon engine (dashed line) and with a mass engine (solid line) as a function of the mass defect coefficient ϵ.

The solid line in Fig. 2 shows the terminal velocity ratio $\omega_{\mathrm{m},\infty}$ of a matter engine as a function of ϵ.

For $0 < \epsilon < 1$, the matter engine is superior to the photon engine and we find terminal velocity ratios $\omega_{\mathrm{m},\infty} \geq \omega_{\mathrm{p},\infty}$, which are smaller than 1. Furthermore, any ramjet that is subject to friction, i.e., absorbs momentum from the incoming mass, will not accelerate endlessly.

V. IDEAL BUSSARD RAMJETS

We define the ideal Bussard ramjet as follows. All the collected cosmic matter is funneled through the spacecraft such that no momentum and energy of the incoming mass is deposited in the spacecraft and then all energy gained by the nuclear fusion of the collected cosmic matter is used for propulsion. This means that the transitions from state 1 to state 2 in the energy momentum diagram in Fig. 1 are vertical transitions, i.e., parallel to the energy axis and without friction.

A. Ideal photon engine

For an ideal Bussard ramjet with photon engine the energy balance of Eq. (14) holds. The momentum equation written without the deceleration term due to the stopping of cosmic matter, $-\mathrm{d}m\omega_1 c$, in Eq. (15) is

$$p_3 = p_1 + p_\mathrm{p}, \tag{22}$$

where p_p is the photon momentum transferred to the ramjet. With $p_\mathrm{p} = E_\mathrm{p}/c$ and Eq. (22) we get from Eq. (14)

$$(\gamma_3 - \gamma_1)\, M - \epsilon\, \mathrm{d}m + M(\omega_3 - \omega_1) = 0. \tag{23}$$

In the differential limit, with $\gamma_3 - \gamma_1 = \mathrm{d}\gamma = \omega\, \mathrm{d}\omega/\gamma$ and $\omega_3 - \omega_1 = \mathrm{d}\omega$, the acceleration [Eq. (6)] of an ideal ramjet with a photon engine is then

$$a_\mathrm{p} = \epsilon \frac{\omega}{\omega + \sqrt{1+\omega^2}} \frac{A\rho c^2}{M}, \tag{24}$$

which is shown in Fig. 3. For the terminal acceleration ($\omega \to \infty$), we obtain

$$a_{\mathrm{p},\infty} = \frac{\epsilon}{2} \frac{A\rho c^2}{M}. \tag{25}$$

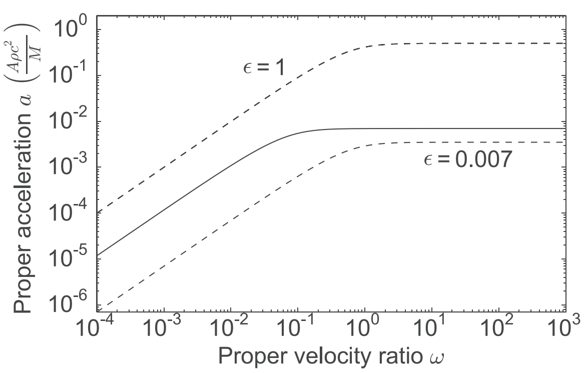

Fig. 3. Achievable acceleration of an ideal Bussard ramjet in units of $A\rho c^2/M$ with radiation engines (dashed lines) and with a matter engine (a solid line) as a function of the velocity ratio ω for $\epsilon = 0.007$ and 1. Note the degeneracy of a mass engine and a photon engine for $\epsilon = 1$.

B. Ideal mass engine

The energy balance of Eq. (18) holds and the momentum balance without the deceleration term due to the stopping of the cosmic matter, $-\mathrm{d}mw_1$, in Eq. (20) becomes

$$p_4 = p_1 + p_\mathrm{m}, \tag{26}$$

where $p_\mathrm{m} = (1-\epsilon)\, \mathrm{d}m\, \omega_\mathrm{m} c$ is the exhaust momentum transferred to the ramjet. With Eq. (18) and the kinetic energy of the exhausted mass we get

$$M(\gamma_4 - \gamma_1) - \epsilon\, \mathrm{d}m + (1-\epsilon)(\gamma_\mathrm{m} - 1)\, \mathrm{d}m = 0. \tag{27}$$

In the differential limit, with $\gamma_4 - \gamma_1 = \mathrm{d}\gamma = \omega\, \mathrm{d}\omega/\gamma$, we get for the acceleration a_m [Eq. (6)] of an ideal ramjet with a matter engine the implicit equation

$$Ma_\mathrm{m} + \gamma_\mathrm{m}(1-\epsilon)A\rho\, c^2 = A\rho\, c^2. \tag{28}$$

With p_m from Eq. (26), γ_m can be eliminated from Eq. (28). This yields a quadratic equation for the acceleration of an ideal matter engine a_m,

$$M^2 a_\mathrm{m}^2 + 2A\rho\, w^2 M a_\mathrm{m} - (A\rho w\, c)^2 \epsilon(2-\epsilon) = 0, \tag{29}$$

with the physical solution

$$a_\mathrm{m} = \omega\left[\sqrt{\omega^2 + \epsilon(2-\epsilon)} - \omega\right] \frac{A\rho\, c^2}{M}. \tag{30}$$

This solution gives the achievable acceleration for every velocity (Fig. 3). For high relativistic velocities we find, with $\sqrt{1-\epsilon(2-\epsilon)/\omega^2} \approx 1 - \epsilon(2-\epsilon)/2\,\omega^2$, the terminal acceleration

$$a_{\mathrm{m},\infty} = \frac{\epsilon(2-\epsilon)}{2} \frac{A\rho\, c^2}{M}. \tag{31}$$

The solution for $\epsilon = 1$ coincides with that of the photon engine, though this is a mathematical limit only, since in this case no matter may be accelerated by the thrust.

VI. CONSEQUENCES OF RELATIVISTIC SPACE TRAVEL

One of the most obvious consequences of relativistic space travel is the detuning of the clocks in the spacecraft and that in the frame of reference of the cosmos. Here, we use the travel times as experienced by the crew on board a relativistic spaceship and see that it is, in principle, possible to cross the universe with an ideal Bussard ramjet in a human lifespan. Furthermore, we investigate how difficult it would be to navigate or change direction of such a vehicle.

A. Travel times

To obtain a terminal acceleration of about g or $10\,\mathrm{m/s^2}$ with $\epsilon = 0.007$, we chose the parameters $A = 1.5 \times 10^{14}\ \mathrm{m}^2$, which is about the size of Earth, $M = 10^8\,\mathrm{kg}$, about the weight of a modern carrier ship or about the weight of a sheet of a monolayer graphene with the diameter of Earth, and $\rho = 10^{-20}\,\mathrm{kg/m^3}$, which corresponds to about 6 hydrogen atoms per cubic centimeter. This assumption on the density is somewhat arbitrary. In dense interstellar clouds,[18] it can amount to 1000 H atoms/cm^3, in the local interstellar medium[19] it is about 5×10^{-3} H atoms/cm^3, or if we consider the average baryon density of 0.047 the critical density of $1-3 \times 10^{-26}$ kg/m^3 in the universe[20] we would deal with a density of about 3×10^{-7} protons/cm^3. This variation of proton density by many orders of magnitude indicates again how difficult it would be to build a ramjet. However, with the above assumption on the density we compute a flight with an ideal Bussard ramjet with a mass engine from a low starting velocity to the terminal acceleration at the end with extremely high relativistic velocities.

The story in "Tau Zero"[1] depends on the fact that with a constant acceleration of 10 m/s^2 a big part of the visible universe can be crossed within a few years. However, the Bussard ramjet does not accelerate quickly at low velocities (see Fig. 3). Since the achievable acceleration a is a known function of the momentary velocity ratio ω', the time required to reach a given velocity ω starting with ω_0 can be found by integration of Eq. (6),

$$\tau(\omega) = c \int_{\omega_0}^{\omega} \frac{d\omega'}{\gamma a(\omega')}, \tag{32}$$

and the distance d in the frame of the cosmos is found by further integration,

$$d(\tau) = c \int_0^{\tau} \omega(\tau')\, d\tau'. \tag{33}$$

In principle, these integrals can be solved analytically, however, their form is so complicated that not much can be learned from them. Therefore, we use a numerical method and plot the acceleration, velocity, and distance, all as functions of time (Fig. 4).

In order to accelerate, the ramjet needs an initial velocity. If the ramjet were constructed in an orbit like Earth's orbit around the sun, an initial velocity of $\omega_0 c = 10^{-4} c$ (30 km/s) may be assumed. With the above parameters, the spacecraft starts slowly for about one quarter of a year, then the acceleration increases quickly and reaches the limit of $0.95g$ already after one year. After about 3.5 years, a flight distance of one light year is reached. After about one year the terminal acceleration is reached and both velocity and distance traveled increase exponentially with time.

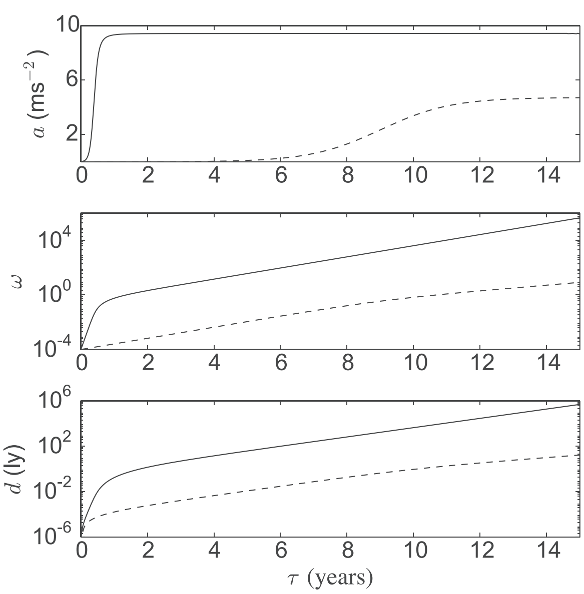

Fig. 4. The first 15 years of the journey of ideal Bussard ramjets as described in the text with a terminal acceleration of $1\,g$ and $\epsilon = 0.007$: acceleration (top panel), velocity (middle panel), and distance traveled in units of light years (bottom panel) as a function of time in years. Solid lines: mass engine; dashed lines: photon engine.

B. Curved trajectories

To obtain a higher relativistic velocity in short time, the engineers in the Bussard ramjet in the novel "Tau Zero" plotted a course of the ship through the center of a galaxy, where the density of cosmic matter is higher:[1]

Their results indicate we can swing halfway around the galaxy, spiraling inward till we plunge straight through its middle and out again on this side. We'd be slow about any course change anyway. (Poul Anderson, "Tau Zero," Chapter 10).

Indeed, navigation requires a change in direction of the motion. To achieve this, thrust must be diverted in a transverse direction with respect to the momentary flight direction. Thus, we need to complement the definition of an ideal Bussard ramjet by its ability to redirect the full possible thrust at some angle to the momentary flight direction.

The achievable radius for the curve is given in Eq. (8)

$$r = \frac{(\omega c)^2}{a}. \tag{34}$$

If the thrust is diverted by 90° by some mirror, it can be fully used to fly a curve without losing forward velocity. Thus, the achievable radius for a photon engine with Eq. (24) is

$$r_p = \frac{\left(\omega + \sqrt{1+\omega^2}\right)\omega}{\epsilon} \frac{M}{A\rho}, \tag{35}$$

and for a mass engine with Eq. (30),

$$r_m = \frac{\omega}{\sqrt{\omega^2 + \epsilon(2-\epsilon)} - \omega} \frac{M}{A\rho}. \tag{36}$$

Figure 5 shows the smallest achievable radius r as a function of the velocity. The flight plan indicated in the quotation from the novel "Tau Zero" at the beginning of this section is

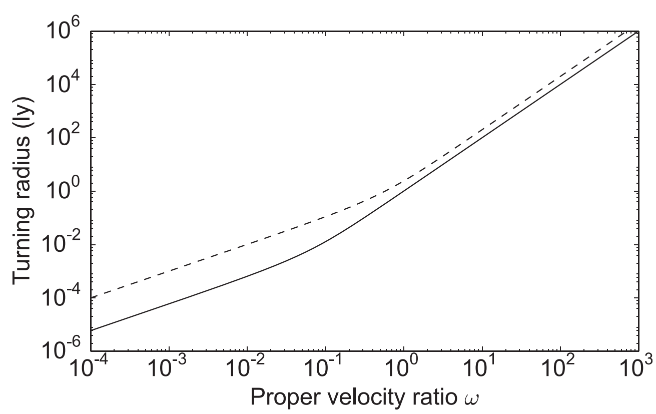

Fig. 5. Minimum possible radius of a curve flown by an ideal Bussard ramjet with $\epsilon = 0.007$. Solid line: mass engine; dashed line: photon engine.

indeed unrealistic. Already for $\omega = 1000$, the turning radius exceeds the diameter of a typical galaxy. However, at such velocities, the Lorentz factor is $\gamma \approx 1000$, thus a trip around the galaxy takes about 3×10^5 years in cosmic time and about 300 years time in the ship.

VII. DISCUSSION AND CONCLUSIONS

A Bussard ramjet is a spacecraft that collects cosmic matter and uses it for propulsion by fusion. We distinguished two engine scenarios for the thrust: propulsion by photon emission or by acceleration of massive particles. It turns out that the mass engine produces higher thrust as long as not all collected cosmic matter is transformed into energy for propulsion. Such a complete mass defect ($\epsilon = 1$) is only expected for matter/antimatter annihilation and would otherwise violate baryon number conservation. It therefore appears unlikely to ever be realized. If the cosmic matter that is used for propulsion is stopped on the ramjet before fusion, the terminal velocity may not exceed one light year per year in the cockpit of the ramjet. Any friction of the ramjet will impose a terminal velocity. This applies as well for absorption of cosmic background radiation[21] interaction with dark matter or generation of gravitational waves. We nevertheless treat the scenario of an ideal, frictionless ramjet that is shown to assume endless acceleration.

If the ramjet uses fusion of protons for propulsion, it needs to collect them in an area of Earth size or more because the density is so small. The energy and momentum balances are written in proper quantities, i.e., in quantities that are felt by the passengers of such a ship. As in the formulation with coordinate quantities,[14] a solution of a quadratic equation gives the possible proper acceleration as a function of proper velocity. Even low velocities, such as that of Earth in orbit around the sun, allow such an ideal ramjet to accelerate and reach relativistic velocities within one proper year and reach a distance of one light year within 3.5 proper years. Once the craft reaches the terminal proper acceleration, the proper velocity and the distance traveled increase exponentially.

Navigation requires change of direction, though it is shown that the maneuverability of a relativistic spacecraft is rather limited. The radius of a possible curve increases rapidly and after two proper years reaches one light year. This makes it impossible to turn within the milky way as suggested in Tau Zero, either because it would take much too long or the turning radius becomes far too large.

We have not discussed the feasibility of a Bussard ramjet. However, we are convinced that it is a pedagogical opportunity to treat the system of equations and use the proper quantities, time, velocity and acceleration, to give relativity a more intuitive appearance, such as experienced by the astronauts in a relativistic spaceship.

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